EXTENSION OF THE ν-METRIC: THE H^∞ CASE

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Abstract. An abtract ν-metric was introduced by Ball and Sasane, with a view towards extending the classical ν-metric of Vinnicombe from the case of rational transfer functions to more general nonrational transfer function classes of infinite-dimensional linear control systems. In this short note, we give an additional concrete special instance of the abstract ν-metric, by verifying all the assumptions demanded in the abstract set-up. This example links the abstract ν-metric with the one proposed by Vinnicombe as a candidate for the ν-metric for nonrational plants.

1. Introduction

We recall the general *stabilization problem* in control theory. Suppose that R is a commutative integral domain with identity (thought of as the class of stable transfer functions) and let $\mathbb{F}(R)$ denote the field of fractions of R. The stabilization problem is:

Given $P \in (\mathbb{F}(R))^{p\times m}$ (an unstable plant transfer function),
find $C \in (\mathbb{F}(R))^{m\times p}$ (a stabilizing controller transfer function),
such that (the closed loop transfer function)

$$H(P,C) := \begin{bmatrix} P \\ I \end{bmatrix} (I-CP)^{-1} \begin{bmatrix} -C & I \end{bmatrix}$$

belongs to $R^{(p+m)\times(p+m)}$ (is stable).

In the *robust stabilization problem*, one goes a step further. One knows that the plant is just an approximation of reality, and so one would really like the controller C to not only stabilize the *nominal* plant P_0, but also all sufficiently close plants P to P_0. The question of what one means by "closeness" of plants thus arises naturally.

So one needs a function d defined on pairs of stabilizable plants such that

(1) d is a metric on the set of all stabilizable plants,
(2) d is amenable to computation, and
(3) stabilizability is a robust property of the plant with respect to this metric.

Such a desirable metric, was introduced by Glenn Vinnicombe in [7] and is called the *ν-metric*. In that paper, essentially R was taken to be the rational functions without poles in the closed unit disk or, more generally, the disk algebra, and the most important results were that the ν-metric is indeed a metric on the set of stabilizable plants, and moreover, one has the inequality that if $P_0, P \in \mathbb{S}(R,p,m)$, then

$$\mu_{P,C} \geq \mu_{P_0,C} - d_\nu(P_0,P),$$

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where $\mu_{P,C}$ denotes the *stability margin* of the pair (P, C), defined by

$$\mu_{P,C} := \|H(P, C)\|_{\infty}^{-1}.$$

This implies in particular that stabilizability is a robust property of the plant P.

The problem of what happens when R is some other ring of stable transfer functions of infinite-dimensional systems was left open in [7]. This problem of extending the ν-metric from the rational case to transfer function classes of infinite-dimensional systems was addressed in [1]. There the starting point in the approach was abstract. It was assumed that R is any commutative integral domain with identity which is a subset of a Banach algebra S satisfying certain assumptions, labelled (A1)-(A4), which are recalled in Section 2. Then an "abstract" ν-metric was defined in this setup, and it was shown in [1] that it does define a metric on the class of all stabilizable plants. It was also shown there that stabilizability is a robust property of the plant.

In [7], it was suggested that the ν-metric in the case when $R = H^{\infty}$ might be defined as follows. Let P_1, P_2 be unstable plants with the normalized left/right coprime factorizations

$$\begin{aligned} P_1 &= N_1 D_1^{-1} = \widetilde{D}_1^{-1} \widetilde{N}_1, \\ P_2 &= N_2 D_2^{-1} = \widetilde{D}_2^{-1} \widetilde{N}_2, \end{aligned}$$

where $N_1, D_1, N_2, D_2, \widetilde{N}_1, \widetilde{D}_1, \widetilde{N}_2, \widetilde{D}_2$ are matrices with H^{∞} entries. Then

$$d_{\nu}(P_1, P_2) = \begin{cases} \|\widetilde{G}_2 G_1\|_{\infty} & \text{if } T_{G_1^* G_2} \text{ is Fredholm with Fredholm index } 0, \\ 0 & \text{otherwise} \end{cases} \tag{1.1}$$

Here $\cdot^*$ has the usual meaning, namely: $G_1^*(\zeta)$ is the transpose of the matrix whose entries are complex conjugates of the entries of the matrix $G_1(\zeta)$, for $\zeta \in \mathbb{T}$. Also in the above, for a matrix $M \in (L^{\infty})^{p \times m}$, $T_M : (H^2)^m \to (H^2)^p$ denotes the *Toeplitz operator* given by

$$T_M \varphi = P_{(H^2)^p}(M\varphi) \quad (\varphi \in (H^2)^m)$$

where $M\varphi$ is considered as an element of $(L^2)^p$ and $P_{(H^2)^p}$ denotes the canonical orthogonal projection from $(L^2)^p$ onto $(H^2)^p$.

Although we are unable to verify whether there is a metric d_{ν} such that the above holds in the case of H^{∞}, we show that the above does work for the somewhat smaller case when R is the class QA of quasicontinuous functions analytic in the unit disk. We prove this by showing that this case is just a special instance of the abstract ν-metric introduced in [1].

The paper is organized as follows:

1. In Section 2, we recall the general setup and assumptions and the abstract metric d_{ν} from [1].
2. In Section 3, we specialize R to a concrete ring of stable transfer functions, and show that our abstract assumptions hold in this particular case.

2. Recap of the abstract ν-metric

We recall the setup from [1]:

- (A1) R is commutative integral domain with identity.
- (A2) S is a unital commutative complex semisimple Banach algebra with an involution $\cdot^*$, such that $R \subset S$. We use inv S to denote the invertible elements of S.

(A3) There exists a map $\iota : \text{inv } S \to G$, where $(G, +)$ is an Abelian group with identity denoted by $\circ$, and ι satisfies
  (I1) $\iota(ab) = \iota(a) + \iota(b)$ $(a, b \in \text{inv } S)$.
  (I2) $\iota(a^*) = -\iota(a)$ $(a \in \text{inv } S)$.
  (I3) ι is locally constant, that is, ι is continuous when G is equipped with the discrete topology.
(A4) $x \in R \cap (\text{inv } S)$ is invertible as an element of R if and only if $\iota(x) = \circ$.

We recall the following standard definitions from the factorization approach to control theory.

The notation $\mathbb{F}(R)$: $\mathbb{F}(R)$ denotes the field of fractions of R.

The notation F^*: If $F \in R^{p\times m}$, then $F^* \in S^{m\times p}$ is the matrix with the entry in the ith row and jth column given by F_{ji}^*, for all $1 \le i \le p$, and all $1 \le j \le m$.

Right coprime/normalized coprime factorization: Given a matrix $P \in (\mathbb{F}(R))^{p\times m}$, a factorization $P = ND^{-1}$, where N, D are matrices with entries from R, is called a *right coprime factorization of* P if there exist matrices X, Y with entries from R such that $XN + YD = I_m$. If moreover it holds that $N^*N + D^*D = I_m$, then the right coprime factorization is referred to as a *normalized* right coprime factorization of P.

Left coprime/normalized coprime factorization: A factorization $P = \widetilde{D}^{-1}\widetilde{N}$, where $\widetilde{N}, \widetilde{D}$ are matrices with entries from R, is called a *left coprime factorization of* P if there exist matrices $\widetilde{X}, \widetilde{Y}$ with entries from R such that $\widetilde{N}\widetilde{X} + \widetilde{D}\widetilde{Y} = I_p$. If moreover it holds that $\widetilde{N}\widetilde{N}^* + \widetilde{D}\widetilde{D}^* = I_p$, then the left coprime factorization is referred to as a *normalized* left coprime factorization of P.

The notation $G, \widetilde{G}, K, \widetilde{K}$: Given $P \in (\mathbb{F}(R))^{p\times m}$ with normalized right and left factorizations $P = ND^{-1}$ and $P = \widetilde{D}^{-1}\widetilde{N}$, respectively, we introduce the following matrices with entries from R:

$$G = \begin{bmatrix} N \\ D \end{bmatrix} \quad \text{and} \quad \widetilde{G} = \begin{bmatrix} -\widetilde{D} & \widetilde{N} \end{bmatrix}.$$

Similarly, given $C \in (\mathbb{F}(R))^{m\times p}$ with normalized right and left factorizations $C = N_C D_C^{-1}$ and $C = \widetilde{D}_C^{-1}\widetilde{N}_C$, respectively, we introduce the following matrices with entries from R:

$$K = \begin{bmatrix} D_C \\ N_C \end{bmatrix} \quad \text{and} \quad \widetilde{K} = \begin{bmatrix} -\widetilde{N}_C & \widetilde{D}_C \end{bmatrix}.$$

The notation $\mathbb{S}(R, p, m)$: We denote by $\mathbb{S}(R, p, m)$ the set of all elements $P \in (\mathbb{F}(R))^{p\times m}$ that possess a normalized right coprime factorization and a normalized left coprime factorization.

We now define the metric d_ν on $\mathbb{S}(R, p, m)$. But first we specify the norm we use for matrices with entries from S.

Definition 2.1 ($\|\cdot\|$)**.** Let $\mathfrak{M}$ denote the maximal ideal space of the Banach algebra S. For a matrix $M \in S^{p\times m}$, we set

$$\|M\| = \max_{\varphi \in \mathfrak{M}} \left\lvert \mathbf{M}(\varphi) \right\rvert. \tag{2.1}$$

Here $\mathbf{M}$ denotes the entry-wise Gelfand transform of M, and $\|\cdot\|$ denotes the induced operator norm from $\mathbb{C}^m$ to $\mathbb{C}^p$. For the sake of concreteness, we fix the standard Euclidean norms on the vector spaces $\mathbb{C}^m$ to $\mathbb{C}^p$.

The maximum in (2.1) exists since $\mathfrak{M}$ is a compact space when it is equipped with Gelfand topology, that is, the weak-$*$ topology induced from $\mathcal{L}(S;\mathbb{C})$. Since we have assumed S to be semisimple, the Gelfand transform

$$\widehat{\cdot} : S \to \widehat{S}\ (\subset C(\mathfrak{M}, \mathbb{C}))$$

is an isomorphism. If $M \in S^{1\times 1} = S$, then we note that there are two norms available for M: the one as we have defined above, namely $\|M\|$, and the norm $\|\cdot\|_S$ of M as an element of the Banach algebra S. But throughout this article, we will use the norm given by (2.1).

Definition 2.2 (Abstract ν-metric d_ν)**.** For $P_1, P_2 \in \mathbb{S}(R, p, m)$, with the normalized left/right coprime factorizations

$$\begin{aligned} P_1 &= N_1 D_1^{-1} = \widetilde{D}_1^{-1} \widetilde{N}_1, \\ P_2 &= N_2 D_2^{-1} = \widetilde{D}_2^{-1} \widetilde{N}_2, \end{aligned}$$

we define

$$d_\nu(P_1, P_2) := \begin{cases} \|\widetilde{G}_2 G_1\| & \text{if } \det(G_1^* G_2) \in \text{inv } S \text{ and } \iota(\det(G_1^* G_2)) = \circ, \\ 1 & \text{otherwise.} \end{cases} \tag{2.2}$$

The following was proved in [1]:

Theorem 2.3. *d_ν given by (2.2) is a metric on $\mathbb{S}(R, p, m)$.*

Definition 2.4. Given $P \in (\mathbb{F}(R))^{p\times m}$ and $C \in (\mathbb{F}(R))^{m\times p}$, the *stability margin* of the pair (P, C) is defined by

$$\mu_{P,C} = \begin{cases} \|H(P, C)\|_\infty^{-1} & \text{if } P \text{ is stabilized by } C, \\ 0 & \text{otherwise.} \end{cases}$$

The number $\mu_{P,C}$ can be interpreted as a measure of the performance of the closed loop system comprising P and C: larger values of $\mu_{P,C}$ correspond to better performance, with $\mu_{P,C} > 0$ if C stabilizes P.

The following was proved in [1]:

Theorem 2.5. *If $P_0, P \in \mathbb{S}(R, p, m)$ and $C \in \mathbb{S}(R, m, p)$, then*

$$\mu_{P,C} \geq \mu_{P_0,C} - d_\nu(P_0, P).$$

The above result says that stabilizability is a robust property of the plant, since if C stabilizes P_0 with a stability margin $\mu_{P,C} > m$, and P is another plant which is close to P_0 in the sense that $d_\nu(P, P_0) \leq m$, then C is also guaranteed to stabilize P.

3. The ν-metric when $R = QA$

Let H^∞ be the Hardy algebra, consisting of all bounded and holomorphic functions defined on the open unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$.

As was observed in the Introduction, it was suggested in [7] to use (1.1) to define a metric on the quotient ring of H^∞. It is tempting to try to do this by using the general setup of [1] with $R = H^\infty$, $S = L^\infty$ and with ι equal to the Fredholm index of the associated Toeplitz operator. However at this level of generality there is no

guarantee that φ invertible in L^∞ implies that T_φ is Fredholm (and hence ι equal to the Fredholm index of the associated Toeplitz operator is not well-defined on inv S (condition (A3)). However a perusal of the extensive literature on Fredholm theory of Toeplitz operators from the 1970s leads to the choices R equal to the class QA of quasianalytic and S equal to the class QC of quasicontinuous functions as conceivably the most general subalgebras of H^∞ and L^∞ which fit the setup of [1], as we now explain.

The notation QC is used for the C^*-subalgebra of $L^\infty(\mathbb{T})$ of *quasicontinuous* functions:

$$QC := (H^\infty + C(\mathbb{T})) \cap \overline{(H^\infty + C(\mathbb{T}))}.$$

An alternative characterization of QC is the following:

$$QC = L^\infty \cap VMO,$$

where VMO is the class of functions of vanishing mean oscillation [4, Theorem 2.3, p.368].

The Banach algebra QA of analytic quasicontinuous functions is

$$QA := H^\infty \cap QC.$$

We have the following.

In order to verify (A4), we will also use the result given below; see [2, Theorem 7.36].

Proposition 3.1. *If $f \in H^\infty(\mathbb{D}) + C(\mathbb{T})$, then T_f is Fredholm if and only if there exist $\delta, \epsilon > 0$ such that*

$$|F(re^{it})| \geq \epsilon \text{ for } 1-\delta < r < 1,$$

where F is the harmonic extension of f to $\mathbb{D}$. Moreover, in this case the index of T_f is the negative of the winding number with respect to the origin of the curve $F(re^{it})$ for $1-\delta < r < 1$.

Theorem 3.2. *Let*

$$\begin{aligned} R &:= QA, \\ S &:= QC, \\ G &:= \mathbb{Z}, \\ \iota &:= \Big(\varphi(\in \text{inv } QC) \mapsto \text{Fredholm index of } T_\varphi(\in \mathbb{Z})\Big). \end{aligned}$$

Then (A1)-(A4) *are satisfied.*

Proof. Since QA is a commutative integral domain with identity, (A1) holds.

The set QC is a unital ($1 \in C(\mathbb{T}) \subset QC$), commutative, complex, semisimple Banach algebra with the involution

$$f^*(\zeta) = \overline{f(\zeta)} \quad (\zeta \in \mathbb{T}).$$

In fact, QC is a C^*-subalgebra of $L^\infty(\mathbb{T})$. So (A2) holds as well.

[5, Corollary 139, p.354] says that if $\varphi \in$ inv QC, then T_φ is a Fredholm operator. Thus it follows that the map $\iota : \text{inv } QC \to \mathbb{Z}$ given by

$$\iota(\varphi) := \text{Fredholm index of } T_\varphi \quad (\varphi \in \text{inv } QC)$$

is well-defined. If $\varphi, \psi \in \text{inv } QC$, then in particular they are elements of $H^\infty + C(\mathbb{T})$, and so the semicommutator

$$T_{\phi\psi} - T_\phi T_\psi$$

is compact [5, Lemma 133, p.350]. Since the Fredholm index is invariant under compact perturbations (see e.g. [5, Part B, 2.5.2(h)]), it follows that the Fredholm index of $T_{\varphi\psi}$ is the same as that of $T_\phi T_\psi$. Consequently (A3)(I1) holds.

Also, if $\varphi \in \text{inv } QC$, then we have that

$$\begin{aligned} \iota(\varphi^*) &= \iota(\overline{\varphi}) \\ &= \text{Fredholm index of } T_{\overline{\varphi}} \\ &= \text{Fredholm index of } (T_\varphi)^* \\ &= -(\text{Fredholm index of } T_\varphi) \\ &= -\iota(\varphi). \end{aligned}$$

Hence (A3)(I2) holds.

The map sending the a Fredholm operator on a Hilbert space to its Fredholm index is locally constant; see for example [6, Part B, 2.5.1.(g)]. For $\varphi \in L^\infty(\mathbb{T})$, $\|T_\varphi\| \leq \|\varphi\|$, and so the map $\varphi \mapsto T_\varphi : \text{inv } QC \to \text{Fred}(H^2)$ is continuous. Consequently the map ι is continuous from inv QC to $\mathbb{Z}$ (where $\mathbb{Z}$ has the discrete topology). Thus (A3)(I3) holds.

Finally, we will show that (A4) holds as well. Let $\varphi \in H^\infty \cap (\text{inv } QC)$ be invertible as an element of H^∞. Then clearly T_φ is invertible, and so has Fredholm index ind T_φ equal to 0. Hence $\iota(\varphi) = 0$. This finishes the proof of the "only if" part in (A4).

Now suppose that $\varphi \in H^\infty \cap (\text{inv } QC)$ and that $\iota(\varphi) = 0$. In particular, φ is invertible as an element of $H^\infty + C(\mathbb{T})$ and the Fredholm index ind T_φ of T_φ is equal to 0. By Proposition 3.1, it follows that there exist $\delta, \epsilon > 0$ such that $|\Phi(re^{it})| \geq \epsilon$ for $1 - \delta < r < 1$, where Φ is the harmonic extension of φ to $\mathbb{D}$. But since $\varphi \in H^\infty$, its harmonic extension Φ is equal to φ. So $|\varphi(re^{it})| \geq \epsilon$ for $1 - \delta < r < 1$. Also since $\iota(\varphi) = 0$, the winding number with respect to the origin of the curve $\varphi(re^{it})$ for $1 - \delta < r < 1$ is equal to 0. By the Argument principle, it follows that f cannot have any zeros inside $r\mathbb{T}$ for $1 - \delta < r < 1$. In light of the above, we can now conclude that there is an $\epsilon' > 0$ such that $|\varphi(z)| > \epsilon'$ for all $z \in \mathbb{D}$. Thus $1/\varphi$ is in H^∞ with H^∞-norm at most $1/\epsilon'$ and we conclude that φ is invertible as an element of H^∞. Consequently (A4) holds. □

In the definition of the ν-metric given in Definition 2.2 corresponding to Lemma 3.2, the $\|\cdot\|_\infty$ now means the usual $L^\infty(\mathbb{T})$ norm.

Lemma 3.3. *Let $A \in QC^{p\times m}$. Then*

$$\|A\| = \|A\|_\infty := \text{ess.sup}_{\zeta \in \mathbb{T}} \left\| A(\zeta) \right\|.$$

Proof. We have that

$$\begin{aligned} \|A\|_\infty &= \text{ess.sup}_{\zeta\in\mathbb{T}} \left\| A(\zeta) \right\| = \text{ess.sup}_{\zeta\in\mathbb{T}} \sigma_{\max}\Big(A(\zeta)\Big) \\ &= \max_{\varphi \in M(L^\infty(\mathbb{T}))} \widehat{\sigma_{\max}(A)}(\varphi) = \max_{\varphi \in M(L^\infty(\mathbb{T}))} \sigma_{\max}\Big(\widehat{A}(\varphi)\Big) \\ &= \max_{\varphi \in M(QC)} \sigma_{\max}\Big(\widehat{A}(\varphi)\Big) = \max_{\varphi \in M(QC)} \left\| \widehat{A}(\varphi) \right\| = \|A\|. \end{aligned}$$

In the above, the notation $\sigma_{\max}(X)$, for a complex matrix $X \in \mathbb{C}^{p\times m}$, means its largest singular value, that is, the square root of the largest eigenvalue of X^*X (or XX^*). We have also used the fact that for an $f \in QC \subset L^\infty(\mathbb{T})$, we have that

$$\max_{\varphi\in M(L^\infty(\mathbb{T}))} \widehat{f}(\varphi) = \|f\|_{L^\infty(\mathbb{T})} = \max_{\varphi\in M(QC)} \widehat{f}(\varphi).$$

Also, we have used the fact that if $\mu \in L^\infty(\mathbb{T})$ is such that

$$\det(\mu^2 I - A^*A) = 0,$$

then upon taking Gelfand transforms, we obtain

$$\det((\widehat{\mu}(\varphi))^2 I - (\widehat{A}(\varphi))^* \widehat{A}(\varphi)) = 0 \quad (\varphi \in M(L^\infty(\mathbb{T}))),$$

to see that $\widehat{\sigma_{\max}(A)}(\varphi) = \sigma_{\max}(\widehat{A}(\varphi))$, $\varphi \in M(L^\infty(\mathbb{T}))$. $\square$

Finally, our scalar winding number condition

$$\det(G_1^*G_2) \in \text{inv } QC \text{ and Fredholm index of } T_{\det(G_1^*G_2))} = 0$$

is exactly the same as the condition

$$T_{G_1^*G_2} \text{ is Fredholm with Fredholm index } 0$$

in (1.1). This is an immediate consequence of the following result due to Douglas [3, p.13, Theorem 6].

Proposition 3.4. *The matrix Toeplitz operator T_Φ with the matrix symbol $\Phi = [\varphi_{ij}] \in (H^\infty + C(\mathbb{T}))^{n\times n}$ is Fredholm if and only if*

$$\inf_{\zeta\in\mathbb{T}} |\det(\varphi(\zeta))| > 0,$$

and moreover the Fredholm index of T_Φ is the negative of the Fredholm index of $\det \Phi$*.*

Thus our abstract metric reduces to the same metric given in (1.1), that is, for plants $P_1, P_2 \in \mathbb{S}(QA, p, m)$, with the normalized left/right coprime factorizations

$$\begin{aligned} P_1 &= N_1 D_1^{-1} = \widetilde{D}_1^{-1}\widetilde{N}_1, \\ P_2 &= N_2 D_2^{-1} = \widetilde{D}_2^{-1}\widetilde{N}_2, \end{aligned}$$

define

$$d_\nu(P_1, P_2) := \begin{cases} \|\widetilde{G}_2 G_1\|_\infty & \text{if } \det(G_1^*G_2) \in \text{inv } QC \text{ and} \\ & \qquad \text{Fredholm index of } T_{\det(G_1^*G_2)} = 0, \\ 1 & \text{otherwise.} \end{cases} \tag{3.1}$$

Summarizing, our main result is the following.

Corollary 3.5. *d_ν given by* (3.1) *is a metric on $\mathbb{S}(QA, p, m)$. Moreover, if $P_0, P \in \mathbb{S}(QA, p, m)$ and $C \in \mathbb{S}(QA, m, p)$, then*

$$\mu_{P,C} \geq \mu_{P_0,C} - d_\nu(P_0, P).$$

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